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Analytical prediction of regular reflection over rigid porous surfaces in pseudo-steady flows

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An analytical model for solving the flow field associated with regular reflections of straight shock waves over porous layers has been developed. The governing equations of the gas inside the porous material were obtained by simplifying the general macroscopic balance equations which were obtained by an averaging process over a representative elementary volume of the microscopic balance equations as originally done by Bear & Bachmat (1990). The analytical predictions of the proposed model were compared to experimental results of Skews (1992) and Kobayashi, Adachi & Suzuki (1993). Very good to excellent agreement was evident.

1. Introduction

The reflection phenomena of shock waves over different geometries and the interaction phenomena of shock or compaction waves with porous media have received much attention in the past two decades owing to their application to many engineering fields. While the former subject has reached a state where, from an engineering point of view, it is quite well understood and as such has been summarized in a few reviews: Bazhenova, Gvozdeva & Nettleton (1984), Hornung (1986) and Ben-Dor (1988) and in a book, Ben-Dor (1991), the latter (interaction phenomena) are still under intensive investigation by many researchers since they are still far from being understood.

The porous media with which the shock waves interact can, in general, be divided into flexible and rigid materials. A comprehensive study concerning the propagation of planar shock/compaction waves in rigid porous media was initiated a few years ago in the Department of Mechanical Engineering of the Ben-Gurion University of the Negev, Israel. The theoretical approach of this study is summarized in Bear *et al.* (1992), Sorek *et al.* (1992) and Levy *et al.* (1995*b*).

The detailed theoretical model which is described by Levy *et al.* (1995*b*) was simplified by Krilov *et al.* (1995) for the case of an isothermal porous material and by Sorek *et al.* (1995) for the more general non-isothermal case. The latter model was recently solved analytically by Levy *et al.* (1995*a*). The predictions of their analytical model regarding the gaseous phase properties were compared to experimental results which were obtained in Skews' laboratory. Excellent agreement was evident.

In a different study Levy *et al.* (1993*b*) developed another simplified model by which the solid phase properties behind strong compaction waves could be predicted. The analytical predictions were compared with the experimental results of Sandusky & Liddiard (1985) and good to very good agreement was obtained.

Based on the fact that the predictions of our simplified analytical models agreed very well with experimental results regarding the properties of both the gaseous and solid

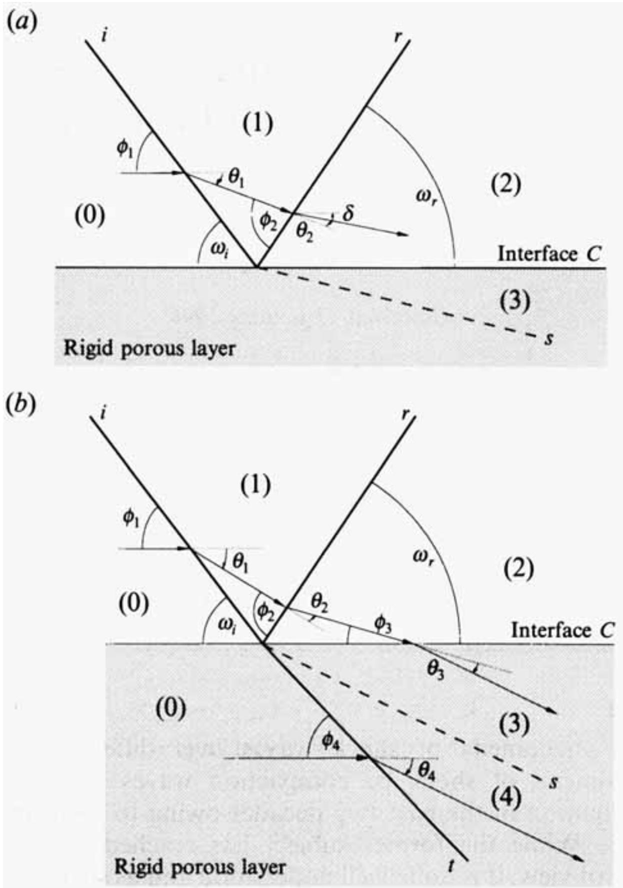

FIGURE 1. Schematic illustration of the two physical models proposed by Kobayashi *et al.* (1993): (*a*) the simple sink model, and (*b*) the realistic model. *i*, *r* and *t* are the incident, reflected and transmitted shock waves, respectively, *s* is a slipstream, *C* is the interface between the porous material and the gaseous phase, ϕ_i and θ_i are angles of incidence and deflection, ω_i and ω_r are the incident and the reflected shock wave angles. (0) to (4) are uniform flow states.

phases behind the shock waves, and our past experience in investigating the shock wave reflection phenomena, we felt confident enough to combine the foregoing mentioned two phenomena and to investigate the regular reflection of oblique shock waves over porous surfaces.

Another motivation for the present study was a paper which was presented recently by Kobayashi, Adachi & Suzuki (1995) at the 19th International Symposium on Shock Waves held in Marseille, France. In that paper, to be discussed in detail subsequently, they proposed two different analytical models for describing the phenomenon. They then presented and solved an analytical model for only the simpler case, where the coupling between the pure gas phase and the porous phase is ignored. Unfortunately, however, this is the less realistic model. As a consequence, we decided to try to solve their second model, the more realistic one, and compare it both with their experimental results, and with those from other experimental investigations.

2. The physical models

The two physical models proposed by Kobayashi *et al.* (1995) are shown schematically in figures 1(*a*) and 1(*b*). The first, shown in figure 1(*a*), was termed by them *the simple sink model.* The incoming flow [state (0)] parallel to the surface of the porous layer is deflected clockwise by an angle θ_1 as it passes across the incident shock wave, *i*. Then, as it passes across the reflected shock wave, *r*, it is deflected counterclockwise by an angle θ_2. Since the flow in state (2) can penetrate into the porous layer, the boundary condition there is

$$\theta_1 - \theta_2 = \delta, \tag{1}$$

where δ is the overall deflection angle induced by the 'sink' effect which is introduced by the porous layer. In the case of a reflection over a non-porous solid surface $\delta = 0$. A similar model was proposed by Onodera & Takayama (1990) who investigated the propagation of planar shock waves over slitted surfaces.

In summary, Kobayashi *et al.*'s (1995) first physical model accounts only for the flow of the gas behind the reflected shock wave into the porous surface and completely ignores the fact that a shock wave should be transmitted into the porous layer as shown schematically in figure 1(*b*), in which their second physical model is shown. This physical model was correctly called by them *the realistic model.* Following the schematic presentation of their realistic model, Kobayashi *et al.* (1995) claimed that 'realistic as it is, it is not easy to solve the whole flow field [associated with this model] since one must deal with the coupled problem between the pure gas phase and the porous phase'.

Their solution of their first physical model was only empirical as they were not able to analytically relate the overall deflection angle, δ, to the initial conditions, namely the incident flow Mach number, M_0, the angle of incidence of the incident shock wave, ϕ_1 and the porous layer properties. Instead, for each experiment they fitted an appropriate value of δ by which the angle between the incident and reflected shock waves, ω_{ir}, as obtained by solving the well-known two-shock theory with the boundary condition given by equation (1) agreed with that obtained experimentally. In the following we present an analytical solution of Kobayashi *et al.*'s (1995) realistic model which deals with the coupled problem between the pure gaseous phase and the porous phase.

3. The present analytical model

As mentioned in the foregoing discussion, the aim of the present study is to formulate the governing equations of the physical model shown in figure 1(*b*), which is most likely the correct model for describing the regular reflection of an oblique shock wave over a rigid porous surface. While the manner of describing the gaseous phase flow through the various shock waves outside the porous layer in states (0), (1) and (2) is relatively simple and well known, the manner of treating the flow inside the porous material is much more complicated and far less known.

The macroscopic mass, linear momentum and energy balance equations, for a two-phase saturated porous medium as obtained by the dimensional analysis which Levy *et al.* (1995*b*) conducted on the macroscopic balance equations of Bear & Sorek (1990) and Sorek *et al.* (1992), were adopted for the present study. The macroscopic conservation equations were obtained by an averaging process over a representative elementary volume (REV) of the microscopic balance equations which were originally presented by Bear & Bachmat (1990). In addition, the porous material was assumed to be rigid and the gaseous phase to be an inviscid, non-conductive perfect gas.

Similar to the modelling of the head-on reflection of a planar shock wave from the shock-tube endwall where both the momentum and energy exchanges between the gaseous phase and the rigid endwall are neglected, it is assumed in the present study that the gaseous phase does not exchange momentum and energy with the rigid skeleton of the porous medium as it flows through the pores. Although this assumption cannot be justified at present, it is expected that it will be validated when the predictions of the presently developed analytical model will be compared with available experimental results. Note that the use of this assumption is limited to the porous region near the interface only. Careful and refined measurements such as those of Adachi, Kobayashi & Suzuki (1992), Kobayashi *et al.* (1995) and Skews (1994) are needed to resolve the issue of how important momentum and energy exchanges are. It should be expected that as the gas propagates deeper into the porous material the momentum and energy exchanges between the gas and the pores become significant. This was shown experimentally by Levy *et al.* (1993*a*) where the shock wave transmitted into the porous material became more and more dispersed as it propagated further and further into the porous material. It should also be noted that this assumption cannot be applied to flexible porous materials even in the region near the interface. This was clearly shown by Olim *et al.* (1994).

In addition to the assumption mentioned above, we adopted the classical assumptions of the well-known two-shock theory (see Ben-Dor 1991) that the flow field is pseudo-steady and all the discontinuities are straight. The last assumption implies that all the flow regions bounded by the straight discontinuities are uniform in all their dynamic and thermodynamic properties.

The justification of these assumptions lies in the recently obtained experimental results of Skews (1994) where measurements of the triple point trajectory indicated that it is 'a straight line within the accuracy of the measurement'. Skews (1994) noted that his findings were in contrast to the evidence of a curved trajectory for rough and guttered wedges as was shown by Reichenbach (1985) and Adachi *et al.* (1992). Note that Onodera & Takayama (1990) who investigated a problem similar to that of Adachi *et al.* (1992) but with a larger ratio of slit width to depth also found that the triple point trajectory was straight. Consequently, if Mach reflection is self-similar, it is expected that the regular reflection, which is simpler, will also be self-similar. (Note that a regular reflection can be considered as a Mach reflection with $\chi \rightarrow 0$ where χ is the triple point trajectory angle.) In addition Skews (1994) noted that in his experiments a number of broad waves were evident in the external flow when the incident shock wave reflected as a Mach reflection. When the reflection was regular (as is the case investigated in the present study) 'the larger scale wave structures were no longer apparent' and the flow behind the reflected shock wave near the reflection point, where our model is applied, approached uniform conditions.

In summary, from a modelling viewpoint it is common practice to treat a rigid porous material as a homogeneous medium with properties uniformly distributed. For such a viewpoint it could be expected that the flow would be pseudo-steady from a macro viewpoint, i.e. it would expand uniformly.

As shown in the Appendix, the properties of the gaseous phase which flows inside the porous layer can be redefined (transformed) to obtain properties which result in governing equations similar to those of a pure gas. Using this novel approach to overall treatment of the phenomena turns out to be much more convenient than the treatment of the original equations.

4. The governing equations

Let us consider the wave configuration shown in figure 1(*b*). The wave configuration consists of three shock waves: the incident shock wave, *i*, the reflected shock wave, *r*, and the transmitted shock wave, *t*; and a contact discontinuity, *s*, which separates the flow which has been shocked by a single shock wave, the transmitted shock wave and the flow shocked by both the incident and reflected shock waves.

State (0) is ahead of the incident and transmitted shock waves; state (1) is behind the incident shock wave; state (2) is behind the reflected shock wave, and state (4) is behind the transmitted shock wave. State (3) is obtained from state (2) when the flow penetrates the porous material. States (3) and (4) are separated by the contact discontinuity.

Applying the conservation equations of mass, normal momentum, tangential momentum and energy across the oblique shock waves which comprise the wave configuration shown in figure 1(*b*) results in:

across the incident shock wave *i*

$$\rho_0 u_0 \sin\phi_1 = \rho_1 u_1 \sin(\phi_1 - \theta_1), \tag{2}$$

$$p_0 + \rho_0 u_0^2 \sin^2\phi_1 = p_1 + \rho_1 u_1^2 \sin^2(\phi_1 - \theta_1), \tag{3}$$

$$\rho_0 \tan\phi_1 = \rho_1 \tan(\phi_1 - \theta_1), \tag{4}$$

$$h_0 + \tfrac{1}{2}u_0^2 \sin^2\phi_1 = h_1 + \tfrac{1}{2}u_1^2 \sin^2(\phi_1 - \theta_1); \tag{5}$$

across the reflected shock wave *r*

$$\rho_1 u_1 \sin\phi_2 = \rho_2 u_2 \sin(\phi_2 - \theta_2), \tag{6}$$

$$p_1 + \rho_1 u_1^2 \sin^2\phi_2 = p_2 + \rho_2 u_2^2 \sin^2(\phi_2 - \theta_2), \tag{7}$$

$$\rho_1 \tan\phi_2 = \rho_2 \tan(\phi_2 - \theta_2), \tag{8}$$

$$h_1 + \tfrac{1}{2}u_1^2 \sin^2\phi_2 = h_2 + \tfrac{1}{2}u_2^2 \sin^2(\phi_2 - \theta_2); \tag{9}$$

across the transmitted shock wave *t*

$$\rho_0^* u_0 \sin\phi_4 = \rho_4^* u_4 \sin(\phi_4 - \theta_4), \tag{10}$$

$$p_0^* + \rho_0^* u_0^2 \sin^2\phi_4 = p_4^* + \rho_4^* u_4^2 \sin^2(\phi_4 - \theta_4), \tag{11}$$

$$\rho_0^* \tan\phi_4 = \rho_4^* \tan(\phi_4 - \theta_4), \tag{12}$$

$$h_0^* + \tfrac{1}{2}u_0^2 \sin^2\phi_4 = h_4^* + \tfrac{1}{2}u_4^2 \sin^2(\phi_4 - \theta_4). \tag{13}$$

(The superscript * denotes that the gaseous phase properties inside the porous layer are redefined as shown in the Appendix.)

In addition to the above equations across the various oblique shock waves, the governing equations across the interface, *C* (which separates the flow inside and outside the porous layer), should be added. Based on Skews (1992) who reported that the wave configurations over porous surfaces were similar to those obtained over slitted surfaces, we assume that the porous surface (near the interface) can be treated as a slitted surface. Thus the governing equations across the interface, *C*, are:

conservation of mass

$$\rho_2 u_2 \sin\phi_3 = \varphi \rho_3^* u_3 \sin(\phi_3 + \theta_3), \tag{14}$$

conservation of normal momentum

$$\varphi p_2 + \rho_2 u_2^2 \sin^2\phi_3 = \frac{\varphi}{\tau} p_3^* + \varphi\rho_3^* u_3^2 \sin^2(\phi_3+\theta_3), \tag{15}$$

conservation of tangential momentum

$$u_2 \cos\phi_3 = u_3 \cos(\phi_3+\theta_3), \tag{16}$$

conservation of energy

$$h_2 + \tfrac{1}{2}u_2^2 \sin^2\phi_3 = \frac{\gamma}{\gamma^*} h_3^* + \tfrac{1}{2}u_3^2 \sin^2(\phi_3+\theta_3), \tag{17}$$

from geometry

$$\phi_3 = \theta_1 - \theta_2. \tag{18}$$

Since the pressures across the contact discontinuity are equal, one can simply write

$$p_3^* = p_4^*. \tag{19}$$

In addition, if the contact discontinuity, s, is assumed to be infinitely thin, i.e. a slipstream, then

$$\theta_1 - \theta_2 + \theta_3 = \theta_4. \tag{20}$$

In the above equations h_i, the enthalpy, can be expressed as

$$h_i = \frac{\gamma}{\gamma-1}\frac{p_i}{\rho_i}. \tag{21a}$$

Similarly

$$h_i^* = \frac{\gamma^*}{\gamma^*-1}\frac{p_i^*}{\rho_i^*}. \tag{21b}$$

The gaseous phase porosity is φ. All the other parameters are defined in the Appendix.

In summary, the above set of governing equations consists of 19 algebraic equations with 25 unknowns, namely ρ_0, ρ_0^*, ρ_1, ρ_2, ρ_3^*, ρ_4^*, p_0, p_0^*, p_1, p_2, p_3^*, p_4^*, u_0, u_1, u_2, u_3, u_4, ϕ_1, ϕ_2, ϕ_3, ϕ_4, θ_1, θ_2, θ_3, and θ_4. Thus, in order to have a solvable set, 6 of the above 25 unknowns should be supplied as initial conditions. The six which are usually the known initial conditions are ρ_0, ρ_0^*, p_0, p_0^*, u_0 and ϕ_1.

Note that the physical properties of both the gaseous and the solid phases, i.e. the specific heat capacities ratio γ, the specific gas constant R, the porosity φ and the tortuosity τ are assumed known.

Numerical solution

The set of the 19 algebraic governing equations was solved numerically by the subroutine DNEQNJ of the IMSL library. This subroutine is designed to solve nonlinear algebraic equations provided the Jacobian is given. The Jacobian was symbolically calculated separately using MATHEMATICA. Since solutions of this kind are extremely fast, no details regarding CPU time are provided here.

5. Results and discussion

In the following, comparisons between predictions of the presently proposed analytical model with the experimental results of Skews (1992) and Kobayashi *et al.* (1995) are given.

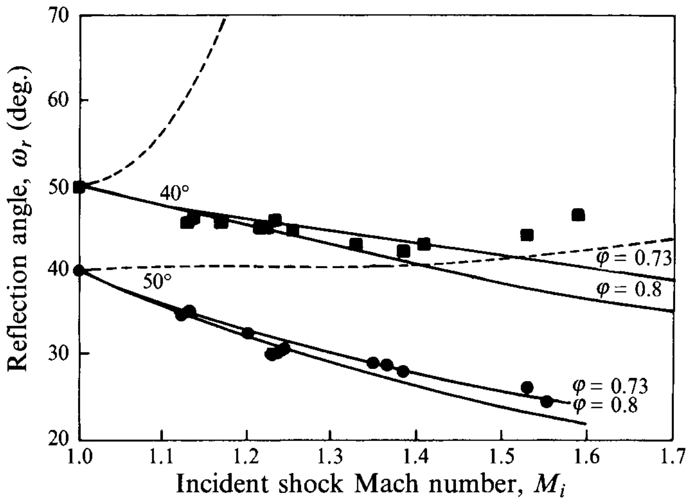

FIGURE 2. The reflected shock wave angle, ω_r, as a function of the incident shock wave Mach number, M_i, for two different reflecting wedge angles, 40° and 50°. The experimental results are taken from Skews (1992). The dashed lines are for plane solid reflecting surfaces taken from Smith (1945). The solid lines are the analytical prediction based on the physical model shown in figure 1(*b*).

Skews (1992) experimentally investigated the oblique reflection of shock waves from porous materials. The porous material in his case was silicone carbide (SiC) matrix with a mean pore size of about 2.5 mm [10 pores per inch (p.p.i.)]. Based on water displacement measurements Skews (1992) estimated the porosity to be 0.8. Levy *et al.* (1993*a*), who used the same material in a different experimental study, calculated the porosity from the approximated expression

$$\varphi = 1 - \rho_c / \rho_s, \tag{22}$$

where ρ_c is the porous material density and ρ_s is the density of the solid material of which the skeleton is made. (ρ_c was obtained by dividing the mass of a given model by its volume and ρ_s was provided by the manufacturer.) The porosity value as obtained and reported by Levy *et al.* (1993*a*) was 0.728 ± 0.016.

Kobayashi *et al.* (1995) conducted experiments with a rubber foam having a porosity 0.98 and a dusty layer whose porosity was reported to be 0.44. Both Skews (1992) and Kobayashi *et al.* (1995) measured the angle, ω_r, between the reflected shock wave, r, and the reflecting porous surface. The values of the reflection angle, ω_r, as a function of the incident shock wave Mach number, M_i, for two different wedge angles, $\theta_w = 40°$ and 50° as measured and reported by Skews (1992), are shown in figure 2 as squares and circles, respectively. The dashed lines describe the dependence between ω_r and M_i as calculated using the two-shock theory for a plane solid reflecting surface. The solid lines were calculated using our proposed analytical model for the two different porosity values mentioned above, i.e. $\varphi = 0.8$ which was reported by Skews (1992) and $\varphi = 0.73$ which was reported by Levy *et al.* (1993*a*). It is evident from figure 2 that our proposed analytical model is capable of reproducing the experimental results very well. Better agreement is obtained with the lower porosity $\varphi = 0.73$ which was measured and reported by Levy *et al.* (1993*a*). Up to $M_i \approx 1.4$ the agreements with both wedges is excellent. It is also evident from figure 2 that for $M_i > 1.4$ much better agreement is obtained with the larger wedge ($\theta_w = 50°$) experiments. The reason for the discrepancy between the experimental results and the analytical prediction for $\theta_w = 40°$ and

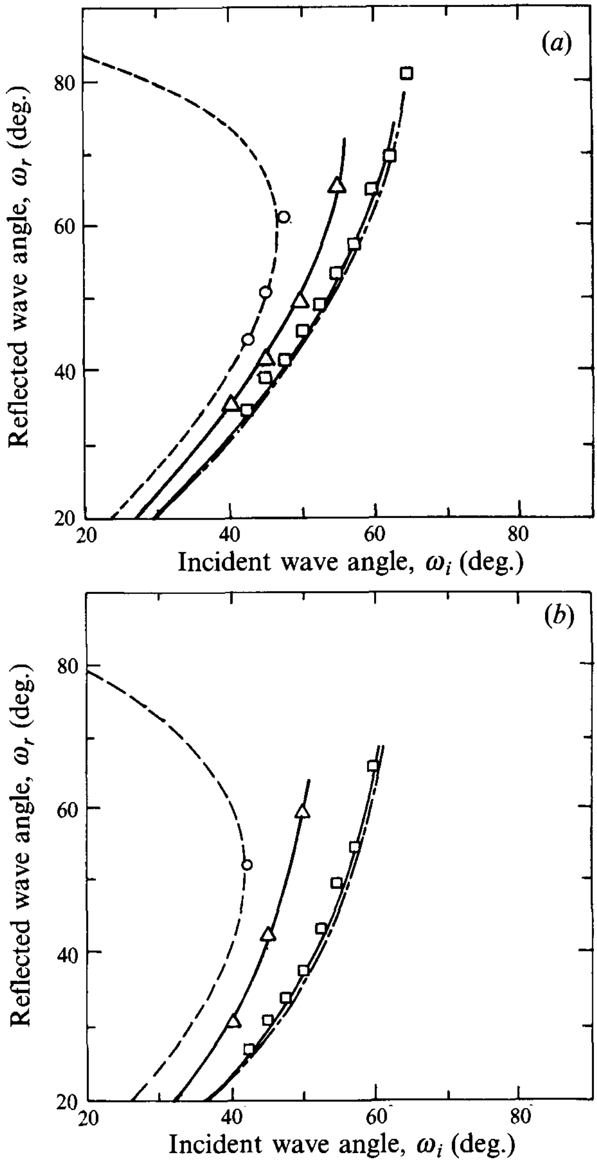

FIGURE 3. The reflected shock wave angle, ω_r, as a function of the incident shock wave angle, ω_i, for two different incident shock wave Mach numbers (*a*) $M_i = 1.20$, (*b*) 1.41. The experimental results are taken from Kobayashi *et al.* (1995). ○, $\varphi = 0$ (plane solid surface); △, $\varphi = 0.44$; □, $\varphi = 0.98$. Dashed line $\varphi = 0$; dashed-dotted line $\varphi = 1$; solid lines $\varphi = 0.44$ and $\varphi = 0.98$ as calculated using the physical model shown in figure 1(*b*).

$M_i > 1.4$ is not known. The discrepancy may arise from one or more of the simplifying assumptions which may have larger effect at smaller wedge angles. (Note that as the wedge angle decreases and the Mach reflection is approached, the disturbances behind the reflected shock wave are much more marked (see Skews 1994).) However, in view of the lack of experimental results for other wedge angles, it was not possible to pinpoint the exact reason for the discrepancy.

Note that unlike the monotonic decrease of the reflection angle, ω_r, with the incident shock wave Mach number, M_i, as obtained experimentally for $\theta_w = 50°$ for the case when $\theta_w = 40°$, ω_r first decreases and then increases. The reason for this behaviour is not clear. The analytical prediction for both these wedge angles resembles a continuous decrease in ω_r as M_i increases.

It should be mentioned here that in order to obtain the above-described analytical prediction, the tortuosity, τ, of the investigated porous material had to be determined. This was done with the aid of equation (A 6*b*) which appears in the Appendix and the experimental results reported by Levy *et al.* (1993*a*) from which the speed of sound of the gas inside the porous layer a^* could be estimated in the following way. Equation (A 6*b*) can be rearranged to read

$$\tau^2 + \frac{1}{\gamma - 1}\tau - \frac{\gamma}{\gamma - 1}\left(\frac{a^*}{a}\right)^2 = 0. \tag{23}$$

Thus, once the value of a^* is known, τ could be simply calculated from equation (23). (Note that (23) yields only one positive root.) The tortuosity as calculated by us using the above outlined procedure and Levy *et al.*'s (1993*a*) experiments was $\tau = 0.7$.

Kobayashi *et al.* (1995) presented their experimentally measured values of the reflection angle, ω_r, as a function of the incident angle, ω_i, for two different incident shock wave Mach numbers, $M_i = 1.2$ and 1.41. Their results are reproduced in figures 3(*a*) and 3(*b*), respectively. The dashed line in each of these figures describes the relation between ω_r and ω_i as obtained from the two-shock theory for reflection over solid plane walls. Note that such a wall could be regarded as a porous material for which $\varphi = 0$. The dashed-dotted line in each of these figures describes the other limiting case of $\varphi = 1$. Details regarding this case can be found in Kobayashi *et al.* (1995).

Naturally, results over any actual porous material for which $0 < \varphi < 1$ should lie in between these two limiting lines as is so in the cases shown in figures 3(*a*) and 3(*b*). The experimental results marked by open circles were obtained over a solid reflected surface ($\varphi = 0$); the triangles were obtained over a dusty layer having a porosity $\varphi = 0.44$ and the squares were obtained for a rubber foam having a porosity $\varphi = 0.98$. The predictions of our proposed analytical model for $\varphi = 0.44$ and 0.98 are shown in figures 3(*a*) and 3(*b*) by solid lines. Excellent agreement is evident with both porosities and both incident shock wave Mach numbers.

It should again be noted that in order to calculate the analytical results shown in figure 3, the tortuosities of both materials (rubber foam and dust layer) were required. Unlike the porous material made of silicon carbide which was used by Skews (1992), where enough experimental data were available to estimate the tortuosity, we did not have a way of getting a good estimate for the tortuosity of the materials used in the experiments of Kobayashi *et al.* Consequently, based on the fact that the tortuosity was found to be slightly smaller than its porosity in the case of a silicon carbide (0.7 as compared with 0.73), the tortuosity of the dust layer was set at 0.4 (its porosity was 0.44) and that of the rubber foam was set at 0.9 (its porosity was 0.98).

It should be noted here that due to this way of choosing the tortuosity values, the sensitivity of the analytical results for the reflection angel, ω_r, with respect to the tortuosity was checked. The calculations for $\tau = 0.4$ and 0.5 resulted in

$$\frac{|\omega_r(\tau = 0.5) - \omega_r(\tau = 0.4)|}{\omega_r(\tau = 0.4)} < 0.01.$$

Thus it is clear from this sensitivity check that the predicted values of the reflected wave angle, ω_r, do not depend strongly on the tortuosity, τ, and that our analysis is valid.

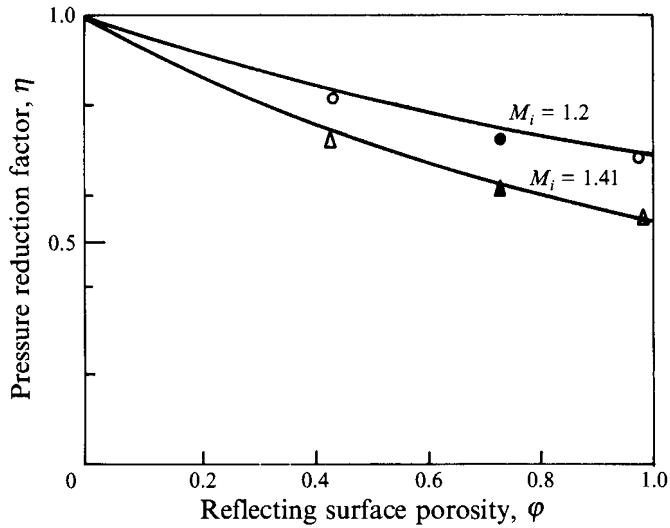

FIGURE 4. The dependence of the pressure reduction factor, $\eta(\varphi) = p_2(\varphi)/p_2(\varphi = 0)$, on the porosity of the reflecting surface, for $\theta_w = 50°$ and comparison with experimental results. ●, ▲, Skews (1992); ○, △, Kobayashi *et al.* (1995).

Owing to the fact that the porous layer over which the shock wave reflects acts as a sink, the pressure in state (2) behind the reflected shock wave is smaller than that obtained when the reflection is over solid surfaces. Consequently, let us define a pressure reduction factor η where

$$\eta(\varphi) = \frac{p_2(\varphi)}{p_2(\varphi = 0)}. \tag{24}$$

The dependence of the pressure reduction factor on the reflecting surface porosity for a reflecting wedge angle $\theta_w = 50°$, is shown in figure 4.

The experimental results of Skews (1992) and Kobayashi *et al.* (1995) for two incident shock wave Mach numbers, $M_i \approx 1.2$ and ≈ 1.4 are added to figure 4. (Note that Skews' experiments were conducted with $M_i = 1.19$ and 1.4 and Kobayashi *et al.*'s with 1.2 and 1.41.) The results for $M_i \approx 1.2$ are marked by circles and those for $M_i \approx 1.4$ by triangles. The experimental results of Skews are marked with solid symbols and those of Kobayashi *et al.* with open symbols. Note that the value of $p_2(\varphi)$ was not presented explicitly by either Skews (1992) or Kobayashi *et al.* (1995). However, based on their reported data of the reflection angle, ω_r, the value of $p_2(\varphi)$ was simply inferred by us using oblique shock wave relations across the incident and reflected shock waves.

The predictions of our proposed analytical model for $M_i = 1.2$ and 1.41 are also shown in figure 4 (solid lines). Very good agreement between the analytical predictions and the experimental results is evident in figure 4 for the porous material with the three different porosities ($\varphi = 0.44$, 0.73 and 0.98). Note that the agreement seems to be better for the higher porosities.

The transition from regular to Mach reflection

The reflecting wedge angle at which the reflection undergoes a transition from regular to Mach reflection, θ_w^{tr}, as a function of the incident shock wave Mach number, M_i, or the inverse pressure ratio, $\xi = p_1/p_0$, is shown in figure 5. Note that the wedge angle, θ_w, complements the incident wave angle, ω_i, i.e. $\omega_i + \theta_w = \pi/2$.

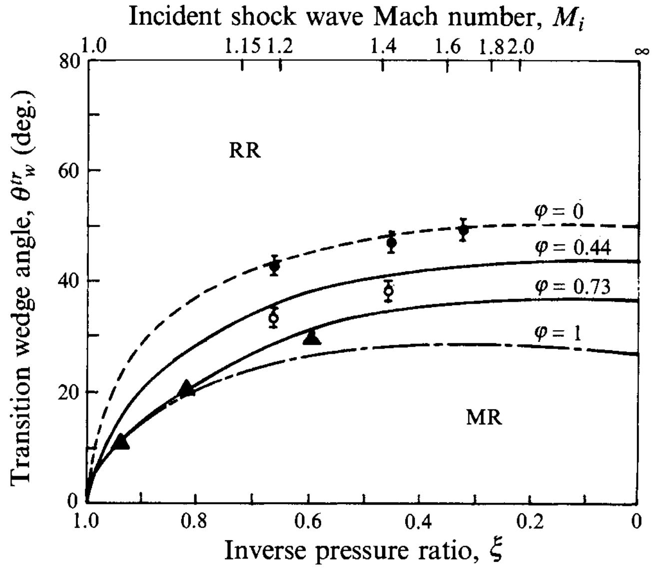

FIGURE 5. The dependence of the regular ⇔ Mach reflection transition on the porosity of the reflecting surface and comparison with the experimental results of Smith (1945) for $\varphi = 0$ (●), Kobayashi *et al.* (1995) for $\varphi = 0.44$ (○), and Skews (1992) for $\varphi = 0.73$ (▲). The curves are calculated using the physical model shown in figure 1(*b*).

Note also, that since the gas behind the reflected shock wave is allowed to penetrate into the porous material, the detachment criterion is meaningless for the present case. Instead, the sonic criterion was applied. It should be mentioned here that for reflection over solid surfaces the sonic and detachment criterion are practically the same. Based on the fact that the disturbances propagate faster above the interface than under it, i.e. in the pure gas as opposed to in the gas inside the porous material, the sonic criterion for the present case is expressed as

$$u_2 \cos \phi_3 = a_2$$

rather than $u_2 = a_2$ which is the sonic criterion over solid non-porous surfaces.

The two limiting lines for $\varphi = 0$ (solid reflecting surface) and $\varphi = 1$ which were mentioned earlier when figure 3 was presented and discussed are shown in figure 5 by dashed and dashed-dotted lines, respectively. Using simple gasdynamic considerations (or sound theory) it could be shown that the line $\varphi = 1$ is given by

$$\theta_w^{tr} = \arctan\left\{\frac{(M_i^2-1)[(\gamma-1)M_i^2+2]}{(\gamma+1)M_i^4}\right\}^{1/2}. \tag{25}$$

The results of Smith (1945) for $\varphi = 0$, Kobayashi *et al.* (1995) for $\varphi = 0.44$ and Skews (1992) for $\varphi = 0.73$ are added to figure 5, together with the analytical predictions of the proposed analytical model which are shown by solid lines.

Note that while in the experiments of Kobayashi *et al.* (1995) the transition wedge angle, θ_w^{tr}, was measured and reported, in Skews' (1992) results the values of θ_w^{tr} were not directly reported. They were deduced by us by extrapolating his experimental data for the triple point trajectory angle, χ, to $\chi = 0$.

The agreement between the analytical predictions and the experimental results is seen to be good. It is somewhat better for the lower incident shock wave Mach number, i.e. $M_i \approx 1.2$, than for $M_i \approx 1.4$.

6. Conclusions

An analytical model for describing regular reflection over porous surfaces has been developed. The very good to excellent agreement which was evident when the analytical predictions of the proposed model were compared with the experimental results of Skews (1992) and Kobayashi *et al.* (1995) validates the various assumptions used by us in the course of developing the analytical model and confirms the validity of the entire physical model. In a future study, concepts similar to those adopted in the present study will be used to develop an analytical model for describing Mach reflections over porous surfaces.

Appendix. The properties of the gaseous phase inside the porous medium and its conservation equations

Based on Bear & Bachmat (1990) the one-dimensional conservation equations of the gaseous phase which flows through the pores of the porous medium are

$$\frac{\partial \boldsymbol{U}}{\partial t}+\boldsymbol{A}\frac{\partial \boldsymbol{U}}{\partial x}=0, \tag{A 1a}$$

where $\boldsymbol{U}$, the dependent variables vector, is

$$\boldsymbol{U}=\begin{bmatrix}\rho\\ u\\ p\end{bmatrix}, \tag{A 1b}$$

and $\boldsymbol{A}$ the coefficients matrix is

$$\boldsymbol{A}=\begin{bmatrix}u & \rho & 0\\ 0 & u & \dfrac{\tau}{\rho}\\ 0 & [1+(\gamma-1)\tau]p & u\end{bmatrix}. \tag{A 1c}$$

In the above equations u is the gaseous phase velocity, ρ is its density, p is its pressure, γ is its specific heat capacities ratio and τ is its tortuosity. The tortuosity is a geometric parameter which for gases is, in general, smaller than unity.

An inspection of the coefficients matrix given by (A 1*c*) indicates that by redefining the thermodynamic properties of the gaseous phase, both the variables vector and the coefficients matrix can be transformed to obtain the forms appropriate to a pure gas, i.e.

$$\boldsymbol{U}^*=\begin{bmatrix}\rho^*\\ u\\ p^*\end{bmatrix} \tag{A 1d}$$

and

$$\boldsymbol{A}^*=\begin{bmatrix}u & \rho^* & 0\\ 0 & u & \dfrac{1}{\rho^*}\\ 0 & \gamma^* p^* & u\end{bmatrix}. \tag{A 1e}$$

The advantage of having the forms given by (A 1*d*) and (A 1*e*) over those given by (A 1*b*) and (A 1*c*) is that the former have analytical solutions which can readily be adapted for the present case. The above-mentioned redefinition is given in the following.

Let us redefine the gaseous phase thermodynamic properties as follows:

$$\left.\begin{aligned} &\rho^* = \rho, \quad p^* = \tau p, \quad T^* = T, \quad \gamma^* = 1+(\gamma-1)\tau, \quad R^* = \tau R, \\ &e^* = \frac{1}{\gamma^*-1}\frac{p^*}{\rho^*} = \frac{1}{\gamma-1}\frac{p}{\rho} = e, \quad h^* = \frac{\gamma^*}{\gamma^*-1}\frac{p^*}{\rho^*} = \frac{\gamma^*}{\gamma}h. \end{aligned}\right\} \tag{A 2}$$

Based on the above redefined gaseous phase properties one can further show that

$$C_v^* = \left.\frac{\partial e^*}{\partial T^*}\right|_{v^*} = \left.\frac{\partial e}{\partial T}\right|_v = C_v, \quad C_p^* = \left.\frac{\partial h^*}{\partial T^*}\right|_{p^*} = \left.\frac{\gamma^*}{\gamma}\frac{\partial h}{\partial T}\right|_p = \frac{\gamma^*}{\gamma}C_p \tag{A 3}$$

(recall that τ is a constant). Combining the above two expressions results in

$$\left.\begin{aligned} &C_p^* - C_v^* = \frac{\gamma^*}{\gamma}C_p - C_v = \frac{\gamma^*}{\gamma}C_p - \frac{1}{\gamma}C_p = \frac{\gamma^*-1}{\gamma}C_p = \tau\frac{(\gamma-1)}{\gamma}C_p = \tau R = R^*, \\ &\frac{C_p^*}{C_v^*} = \frac{\gamma^*/\gamma\, C_p}{C_v} = \gamma^* \end{aligned}\right\} \tag{A 4}$$

Substituting the redefined properties into the conservation equations (A 1*a*) yields

$$\left.\begin{aligned} &\frac{\partial \rho^*}{\partial t} + u\frac{\partial \rho^*}{\partial x} + \rho^*\frac{\partial u}{\partial x} = 0, \\ &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho^*}\frac{\partial p^*}{\partial x} = 0, \\ &\frac{1}{p^*}\frac{\partial p^*}{\partial t} + \gamma^*\frac{\partial u}{\partial x} + \frac{u}{p^*}\frac{\partial p^*}{\partial x} = 0. \end{aligned}\right\} \tag{A 5}$$

The eigenvalues of these equations are

$$u \quad \text{and} \quad u \pm (\gamma^* p^*/\rho^*)^{1/2}.$$

Consequently, the disturbance propagation speed, a^*, is clearly

$$a^* = (\gamma^* p^*/\rho^*)^{1/2}. \tag{A 6a}$$

Note that with the aid of the definitions given by (A 2) we have

$$(a^*)^2 = \frac{\gamma^* p^*}{\rho^*} = \frac{\gamma^*}{\gamma}\tau a^2. \tag{A 6b}$$

Since a^* can be considered as the local speed of sound of the gaseous phase inside the porous medium, the gas Mach number inside the porous medium can be defined as

$$M^* = u/a^*. \tag{A 7}$$

Rewriting the governing equations for a steady flow yields

$$u\frac{\partial \rho^*}{\partial x} + \rho^*\frac{\partial u}{\partial x} = 0, \quad u\frac{\partial u}{\partial x} + \frac{1}{\rho^*}\frac{\partial p^*}{\partial x} = 0, \quad \gamma^*\frac{\partial u}{\partial x} + \frac{u}{p^*}\frac{\partial p^*}{\partial x} = 0. \tag{A 8}$$

Applying these equations across the normal shock wave implies that

$$\rho_1^* u_1 = \rho_2^* u_2, \quad p_1^* + \rho_1^* u_1^2 = p_2^* + \rho_2^* u_2^2, \quad h_1^* + \tfrac{1}{2}u_1^2 = h_2^* + \tfrac{1}{2}u_2^2. \tag{A 9}$$

Recall that the set of equations given by (A 9) was developed for the gaseous phase inside the porous material following a redefinition of its thermodynamic properties.

Since the equations have identical form to those obtained across a normal shock wave in a pure gas, it is clear that their solution should also be identical, i.e. one can readily apply the well-known Rankine–Hugoniot relations across the shock wave inside the porous medium provided that the gas real thermodynamic properties are replaced by the above-defined *-properties.

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